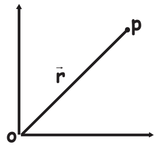
MOTION IN A PLANE

VECTORS

Magnitude Of Vectors

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

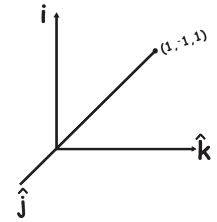
$|\vec{r}| = \sqrt{x^2+y^2+z^2}$



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}|$ or $r = \sqrt{x^2+y^2+z^2}$

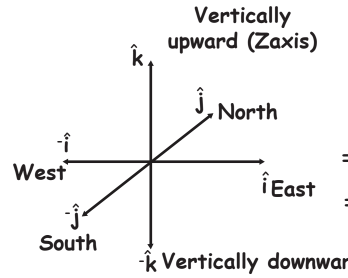
unit vector $= \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$



$\vec{r} = \hat{i} - \hat{j} + \hat{k}$

$|\vec{r}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\hat{r} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

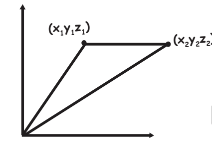


5m north
3m south
2m east

$= 5\hat{j} - 3\hat{j} + 2\hat{i}$

$= 2\hat{j} + 2\hat{i}$

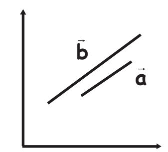
Displacement Vector



$\Delta\vec{r} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$

$|\Delta\vec{r}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Parallel Vectors



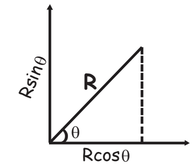
$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

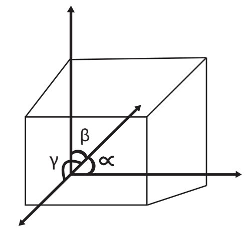
$\vec{a} = m\vec{b}$

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = m$

Components of Vector



$\vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$



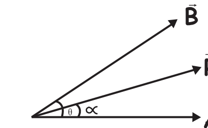
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Addition Of Vectors

$\vec{R} = \sqrt{A^2+B^2+2AB\cos\theta}$

$\vec{R}_{max} = \vec{A} + \vec{B}$ $\quad$ $\vec{R}_{min} = \vec{A} - \vec{B}$



$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$

Vector product

$\vec{C} = \vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}[A_yB_z - B_yA_z] - \hat{j}[A_xB_z - A_zB_x] + \hat{k}[A_xB_y - A_yB_x]$$

Dot product

$x = \vec{A}\cdot\vec{B} = AB\cos\theta$

PROJECTILE MOTION

Projectile motion

Horizontal component $= U\cos\theta$

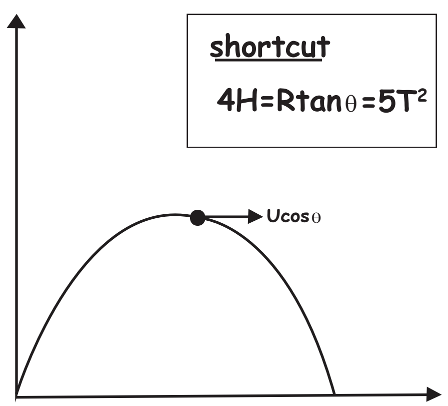
Vertical component $= U\sin\theta$

$$H = \frac{U^2\sin^2\theta}{2g} = \frac{(U\sin\theta)^2}{2g} = \frac{U_y^{\,2}}{2g}$$

$$R = \frac{U^2\sin 2\theta}{g} = \frac{2U\sin\theta\, U\cos\theta}{g} = \frac{2U_xU_y}{g}$$

shortcut

$4H = R\tan\theta = 5T^2$

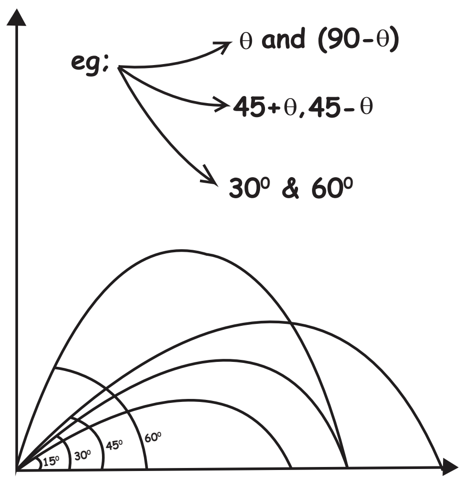


Same range for θ and (90−θ)

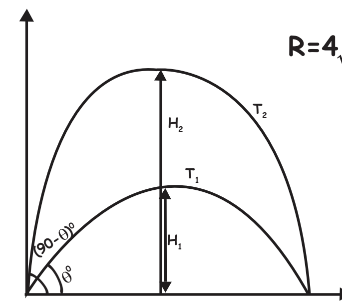
eg; θ and (90−θ)

45+θ, 45−θ

30° & 60°



$R = 4\sqrt{H_1 \cdot H_2}$



- $\frac{T_1}{T_2} = \tan\theta$
- $\frac{H_1}{H_2} = \tan^2\theta$
- $T_1 \times T_2 = \frac{2R}{g}$
- $H_1 \times H_2 = \frac{R^2}{16}$

$T_1^{\,2} + T_2^{\,2} = \frac{4R_{max}}{g}$ $\quad$ $H_1 + H_2 = \frac{R_{max}}{2}$

Maximum range

For $\theta = 45°$

$R_{max} = \frac{U^2}{g}$

From the relation,

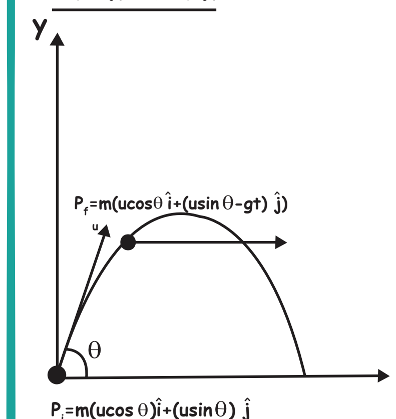
$4H = R\tan\theta = 5T^2$

$4H = R\tan 45$ $\quad$ $H = \frac{R_{max}}{4} = \frac{U^2}{4g}$ $\quad$ $R_{max} = \frac{U^2}{g}$

$4H = R_{max}$

KE at maximum height $= K\cos^2\theta$

Momentum



$P_i = m(u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

$\Delta p = -mgt\,\hat{j}$

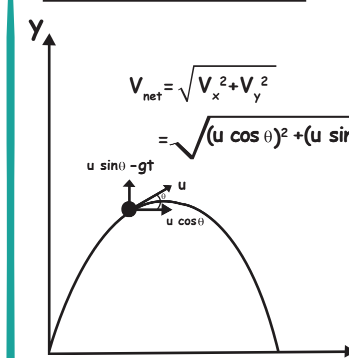
$= -mg \times \frac{u\sin\theta}{g}\hat{j}$

$= -mu\sin\theta\,\hat{j}$

Equation of Velocity

$V_{net} = \sqrt{V_x^{\,2} + V_y^{\,2}}$

$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$



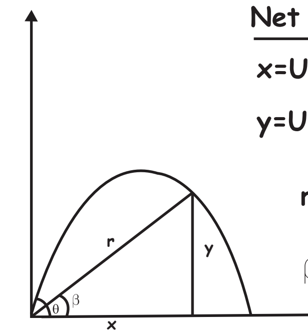
Net displacement

$x = U\cos\theta t$

$y = U\sin\theta t - \frac{1}{2}gt^2$

$r = \sqrt{x^2+y^2}$

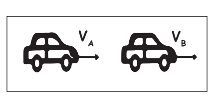
$\beta = \tan^{-1}(y/x)$



RELATIVE MOTION

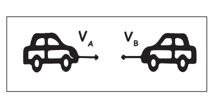
Relative Motion

1) Velocity of A with respect to B $V_{AB} = V_A - V_B$



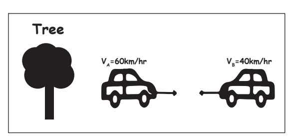
2) $V_{A/B} = V_A - V_B$

$= V_A - (-V_B) = V_A + V_B$



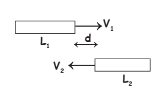
3) $V_{A/Tree} = V_A - V_{Tree} = 60 - 0 = 60$

$V_{B/Tree} = V_B - V_{Tree} = {}^{-}40$

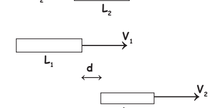


Relative Motion in one dimension overtaking & chasing

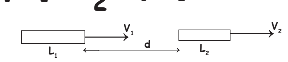
1) $t = \frac{d+L_1+L_2}{V_1+V_2}$



2) $t = \frac{d+L_1+L_2}{V_1-V_2}$



3) $d+L_1+L_2 = (u_1-u_2)t + \frac{1}{2}(a_1-a_2)t^2$



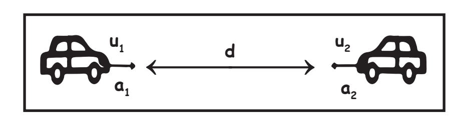
Stopping distance

$u_{rel} = u_1 - u_2$ $\quad$ $a_{rel} = a_1 - a_2$

$0 = u^2 - 2as$

$0 = u^2_{rel} - 2a_{rel}s$

$$\boxed{S = \frac{u^2_{rel}}{2a_{rel}}}$$



MOTION IN A PLANE - 02

RELATIVE MOTION

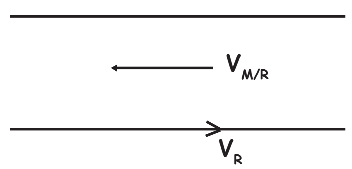
Man-river problem

1) $\vec{V}_{MR}$ or $\vec{V}_{M/Still\ water}$ = velocity due to effort of man [V]/ velocity in still water.

2) V_R = velocity of River

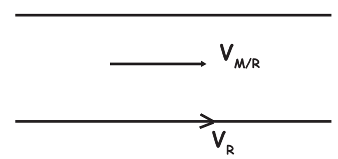
3) V_m = Resultant velocity of man with respect to ground

1) Upstream



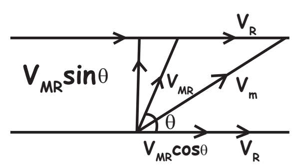
$V_M = V_{M/R} - V_R$

2) Down stream



$V_M = V_{M/R} + V_R$

Swimming across the river



1) $V_\perp = V_{MR}\sin\theta$

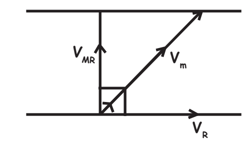
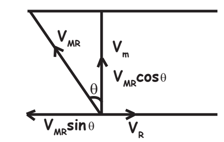
2) $V_{llel} = V_{MR}\cos\theta + V_R$

$t_{cross} = \frac{d}{V_{MR}\cos\theta} = \frac{d}{V_\perp}$

$X_{drift} = (V_{MR}\cos\theta + V_R) \times t$

$= V_{MR}\cos\theta \times t$ + $V_R t$

Due to effort of man — Additional

Shortest time	Shortest Path
	
• $t = \frac{d}{V_{MR}\sin\theta}$; $\theta = 90$, $\sin\theta = 1$	$V_{MR}\sin\theta = V_R$; $V_{MR}\cos\theta = V_m$
• $t_{min} = \frac{d}{V_{MR}}$	Condition for no drifting
• $X_{drift} = V_R \times t$	$\Rightarrow \sin\theta = \frac{V_R}{V_{MR}}$
• $V_m = \sqrt{(V_{MR})^2 + (V_R)^2}$	$\Rightarrow t_{cross} = \frac{d}{V_M}$
	$\Rightarrow V_m = \sqrt{V_{MR}^2 - V_R^2}$
	$\Rightarrow$ Drift=0

Escalator

$$t_3 = \frac{d}{V_E + V_{M/E}} = \frac{d}{\frac{d}{t_1} + \frac{d}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

t_1=Time taken by a man to move distance d on a stationary escalator

t_2=Time taken by a man to move distance d along a moving escalator

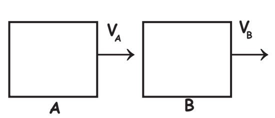
t_3=Time taken by a man to move distance d while walking along a moving escalator

V_E=Velocity of escalator

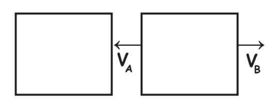
$V_{M/E}$=Velocity of man w.r.t escalator

MAN RAIN PROBLEM

Man-rain problem

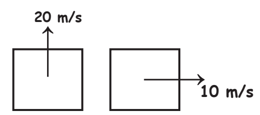


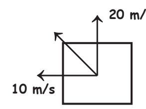
Inorder to find the relative velocity of B with respect to A we have to reverse the direction of vector A and add it with vector B



$V_{B/A} = V_B - V_A$, V_B w.r.t A

$V_{A/B} = V_B - V_A$, V_A w.r.t B





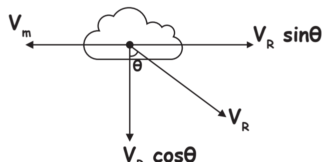
Terms

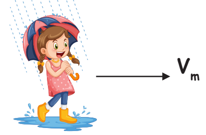
$V_R \Rightarrow$ Velocity of rain w.r.t stationary man

$V_m \Rightarrow$ Velocity of man

$V_{R/m} \Rightarrow$ Velocity of Rain w.r.t man

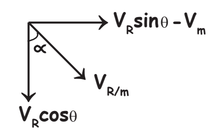
Method





$V_{R/m} = \sqrt{(V_R\sin\theta - V_m)^2 + (V_R\cos\theta)^2}$

Case 1

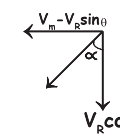


$V_R\sin\theta > V_m$

$\tan\alpha = \frac{V_R\sin\theta - V_m}{V_R\cos\theta}$

$\alpha = \tan^{-1}\left(\frac{V_R\sin\theta - V_m}{V_R\cos\theta}\right)$

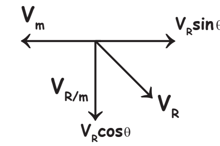
Case 2



$V_m > V_R\sin\theta$

$V_{R/m} = \sqrt{(V_R\cos\theta)^2 + (V_m - V_R\sin\theta)^2}$

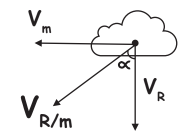
Case 3



$V_m = V_R\sin\theta$

$V_{R/m} = V_R\sin\theta$

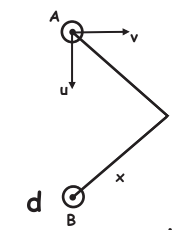
Case 4



$V_{R/m} = \sqrt{V_R^2 + V_m^2}$

$\tan\alpha = \frac{V_m}{V_R}$

Shortest path



shortest path, $x = \frac{d \times V}{\sqrt{V^2 + U^2}}$

Time taken to reach shortest path $= \frac{d \times U}{V^2 + U^2}$

CIRCULAR MOTION

$\boxed{1° = \frac{\pi}{180}}$

Angular velocity

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$

$\vec{V} = \vec{\omega} \times \vec{r}$

$\omega_{sec} = \frac{2\pi}{60} = \frac{2\pi}{T}$

$\omega_{min} = \frac{2\pi}{60 \times 60}$ $\quad \omega_{hour} = \frac{2\pi}{12 \times 60 \times 60}$ $\quad \frac{\omega_{min}}{\omega_{hour}} = \frac{12}{1}$

Angular acceleration

$\alpha = \frac{d\omega}{dt}$ $\quad a_t = r\alpha$

$\vec{a}_t = \vec{\alpha} \times \vec{r}$

Equation of angular motion

1) Constant angular velocity :- ω = constant

2) Constant angular acceleration

$\Rightarrow \omega = \omega_0 + \alpha t$

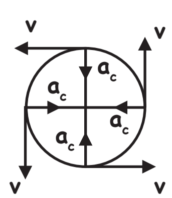
$\Rightarrow \Delta\theta = \omega_0 t + 1/2\,\alpha t^2$

$\Rightarrow \omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

Centripetal acceleration

Directed towards centre

Not a constant vector



-change the direction of velocity

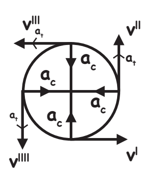
· $a_c = \frac{v^2}{R} = a_c = r\omega^2$

· $\vec{a}_c \perp \vec{v} \longrightarrow$ Power= $\vec{F}.\vec{V} = 0$

· $\vec{F}_c \perp \vec{s} \longrightarrow$ Work= $\vec{F}.\vec{S} = 0$

· $\triangle$K=0

Tangential acceleration



In direction of tangent

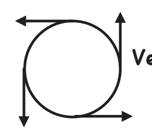
$a_t = \frac{dv}{dt} = r\alpha \longrightarrow \alpha = \frac{d\omega}{dt}$

Tangential Force

$F_t = ma_t$

Resultant acceleration

$$a_r = \sqrt{a_c^2 + a_t^2}$$



Velocity is tangent to the circle

Circular Motion

Uniform Circular Motion

- Speed Constant
- Direction of velocity changes
- ω = Constant
- $a_c = \frac{v^2}{r}, r\omega^2$
- $a_t = 0$

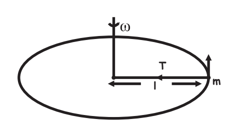
Non-uniform Circular Motion

- Speed not Constant
- Velocity changes in direction and magnitude
- a_c = Centripetal acceleration
- a_t = tangential acceleration

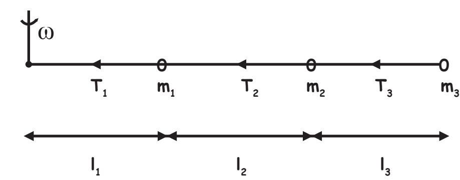
$= \frac{dv}{dt}$ $\quad \alpha = \frac{d\omega}{dt}$

- ω = Changes $\longrightarrow \alpha$ angular acceleration

Horizontal circular motion



$T = \frac{mv^2}{r} = mr\omega^2 = ml\omega^2$

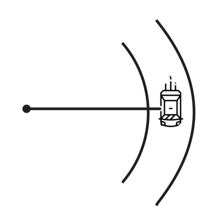


$T_1 = m_1 l_1 \omega^2 + m_2(l_1 + l_2)\omega^2 + m_3(l_1 + l_2 + l_3)\omega^2$

$T_2 = m_2(l_1 + l_2)\omega^2 + m_3(l_1 + l_2 + l_3)\omega^2$

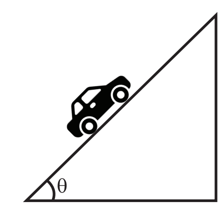
$T_3 = m_3(l_1 + l_2 + l_3)\omega^2$

Flat circular track



$V_{max} = \sqrt{\mu_s Rg}$

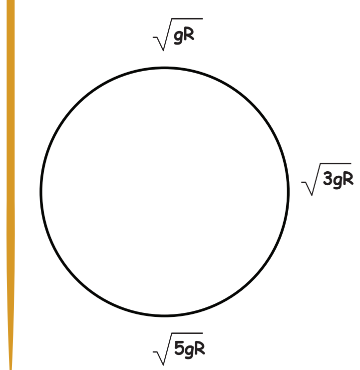
Banking of Road



$V_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$

$V_{opt} = \sqrt{rg\tan\theta}$

$V_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{1 + \mu\tan\theta}}$



At Bottom

a) $T_{max} = \frac{mv^2}{r} + mg$

b) min velocity at bottom of the circle $= \sqrt{5gR}$

At Top

a) $T_{min} = \frac{mv^2}{r} - mg$

b) min velocity at top to complete the circle $= \sqrt{gR}$